
A conservative Eulerian finite element method for transport and diffusion in moving domains

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Abstract

Many mathematical models from biology, chemistry, physics, and engineering are based on partial differential equations posed on evolving domains. A common challenge faced in such models is to build a discretisation which is able to accurately and efficiently deal with both Lagrangian (e.g., displacements) and Eulerian (e.g., temperature, concentrations) quantities. In recent years, unfitted, immersed, or embedded finite element methods have become popular to deal with such problems on geometrically complex and time-dependent domains.

We consider an unfitted finite element method known as CutFEM. Here, we embed the domain into a background domain, which can then be easily meshed. This eliminates the need for remeshing and the projection of the solution between meshes as the domain evolves. Together with ghost-penalty stabilisation, the resulting method is robust with respect to arbitrary intersections between the geometry and mesh.

To discretise the moving domain problem in time, we follow an idea by Lehrenfeld and Olshanskii (ESAIM: M2AN, 53(2): 585-614, 2019). The idea is to remain in a fully Eulerian framework by using the discrete extension of the solution provided by ghost-penalty stabilisation to enable a method-of-line discretisation of the time-derivative $\partial_t \hat{u} = \frac{1}{\Delta t} (\hat{u} - u_{n-1})$, even when the domains Ω_n and Ω_{n-1} do not coincide. With Δt

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